LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2023**

PMT 2501 – ALGEBRA

Date: 09-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Answer ALL questions:

1. a. If $O(G) = p^2$ where p is a prime number then prove that G is abelian. (5)

[OR]

b. Write conjugate classes of S_3 . (5)

c. State and prove second and third part of Syllow's theorems. (15)

[OR]

d. i) State and prove Cauchy's theorem.

ii) Let G be a group and $a \in G$, prove that $N(a)$ is a subgroup of G . (10 + 5)

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2. a. Define internal direct product and give an example. (5)

[OR]

b. State and prove division algorithm. (5)

c. Discuss about the field $\frac{Q[x]}{x^2-2}$. (15)

[OR]

d. i) Prove that product of two primitive polynomial is primitive.

ii) If *f(x)* and *g(x)* are two nonzero polynomials, then prove that

$deg\left(f(x)g(x)\right)=deg\left(f(x)+deg(gx)\right)$. (10 + 5)

3. a. Find the degree of $\sqrt{2}+\sqrt{3}$ over Q. (5)

[OR]

b. Show that $x^5 + 6x^4 + 9x^4 - 12x^2 + 30x + 3$ is irreducible over rationals. (5)

c. If L is the finite extension of K and K is the finite extension of F then prove that L is the finite extension of F. (15)

[OR]

d. Show that a polynomial of degree *n* can have at most *n* roots in any extension field. (15)

4. a. If a, b in K are algebraic over F, then prove that a±b, ab and a/b (if b≠0) are algebraic over F. (5)

[OR]

b. Define Galois group and give an example. (5)

c. The element a ∈ K is said to be algebraic over F if and only if F(a) is a finite extension over F. (15)

[OR]

d. State and prove fundamental theorem of Galois Theory. (15)

5. a. Give an example of a finite field with order 27. (5)

[OR]

b. Show that any finite field has p^m elements where p is a prime number. (5)

c. Prove that the multiplicative group of non-zero elements of a finite field is cyclic. (15)

[OR]

d. Prove that any finite division ring is necessarily a commutative field. (15)

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